

# Design of Flare Systems Using Safety Instrumented Functions

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Published online 30 October 2009 in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/prs.10364

*In the design of a flare header system, it is common practice to design the system for the total load from all the relief valves discharging into the system for a given scenario such as power failure or cooling water failure. The total load can be reduced by the application to the process of one or more safety instrumented functions (SIFs) which are designed to eliminate specific loads. However, full credit for all the SIFs cannot be taken because some of them may fail when called on to operate. Thus, to design the flare system, the designer must assume some of the SIFs fail and the question to be decided is how many SIFs should be assumed to have failed. The purpose of this article is to show how to determine the number of SIFs likely to fail on a given demand. It is shown that the probability of failure is governed by the binomial probability distribution. This probability is independent of system geometry and individual loads. The ideas of the binomial probability distribution are then expanded to show how, by explicit consideration of system geometry and individual loads, the size of the system may be further reduced. © 2009 American Institute of Chemical Engineers Process Saf Prog 29: 166–173, 2010*

*Keywords: flare headers; safety instrumented functions; credit; binomial probability distribution*

## INTRODUCTION

API-Standard 521, *Pressure-relieving and Depressuring Systems* [1], provides guidance on the design of relief valve discharge header systems. In the design of such systems, API allows credit to be taken for the favorable response of instrumentation to reduce the total system load and hence reduce the size of the system. However, API states that the decision as to whether or not to exclude a specific load from the design should consider the number and reliability of the instrumented systems. An approach specifically mentioned is to determine the percentage of

instrumented systems likely to fail and then design the header system to include those loads.

The purpose of this article is to show how to go about determining the percentage of instrumented systems that are likely to fail for the specific case in which all the systems are Safety Instrumented Functions (SIFs) with a given Safety Integrity Level (SIL) and are completely independent. Specifically, this article addresses the following two questions:

1. Given  $N$  independent SIFs with a given SIL, what is the probability that  $k$  or more of the SIFs will fail on demand?
2. Given a relief header configuration with  $N$  independent SIFs, what is the probability that the design parameters of the system (e.g., back pressure on the individual valves, radiation) will be exceeded?

The analysis of this article is only applicable when all of the SIFs are independent. The implementation of the SIFs must therefore involve separate and independent sensors, final elements and logic solvers and there should be no common cause failures between the SIFs. It is essential that any final design be thoroughly reviewed to ensure that this independence requirement is met.

## BACKGROUND ON PROBABILITY THEORY

Consider two events, A and B, which are completely independent of each other and have probabilities of occurring  $P_A$  and  $P_B$ , respectively. On a given trial, the probability of A and B occurring,  $P(A \text{ and } B)$ , is given by  $P_A P_B$ . The probability of A or B occurring,  $P(A \text{ or } B)$ , is given by  $P_A + P_B$ .

For example, if we roll two dice, marked “1” and “2,” a single time, the probability of getting a one on the die marked “1” and a five on the die marked “2” is  $(1/6)(1/6)$ . The probability of getting a five on “1” and a one on “2” is also  $(1/6)(1/6)$ . The probability of getting a (one on “1”/five on “2”) or (five on “1”/one on “2”) is  $(1/6)(1/6) + (1/6)(1/6)$ . Notice this latter combination can be written as  $2(1/6)(1/6)$  which is

the probability of obtaining one “one” and one “five” times the number of ways a one/five combination can appear.

As a second example, consider a single die. The probability of rolling a one is (1/6). The probability of not rolling a one is (5/6). Thus, if we roll two dice, marked “1” and “2,” a single time, the probability of getting a one on the die marked “1” and not getting one on the die marked “2” is (1/6)(5/6). The probability of not getting a one on “1” and getting a one on “2” is (5/6)(1/6). The probability of getting a (one on “1”/not-one on “2”) or (not-one on “1”/one on “2”) is (1/6)(5/6) + (5/6)(1/6). Notice this latter combination can be written as 2(1/6)(5/6) which is the probability of obtaining a “one” and “not-one” times the number of ways a (one/not-one) combination can appear.

The extension to more than two events is straightforward. For example, for  $N$  dice labeled 1,2,.. $n$ ... $N$ , the probability of rolling a one on the die marked 1 and not rolling a one on the remaining dice is (1/6)(5/6) $^{N-1}$ . Similarly, the probability of rolling a one on the die marked “ $n$ ” and not rolling a one on the remaining dice is (1/6)(5/6) $^{N-1}$ . The probability of rolling a one on any die and not rolling a one on the remaining dice is the sum of the individual probabilities which is  $N(1/6)(5/6)^{N-1}$ . Notice that the multiplier  $N$  is the number of ways a (one/not-one) combination can occur.

#### APPLICATION TO SAFETY INSTRUMENTED FUNCTIONS

The SIL for a specific safety instrumented function gives the probability the system will fail on demand. A system rated for SIL-1 will fail no more than once in every 10 demands. Systems rated for SIL-2 or SIL-3 will fail no more than once in every 100 or 1,000 demands, respectively. In terms of probability, the probability,  $P$ , that a SIL-1 safety instrumented function will fail is 0.1. For SIL-2 and SIL-3, the probabilities are 0.01 and 0.001, respectively.

To determine the probability of  $k$  or more SIFs failing, it is useful to consider the specific case of four SIFs ( $N = 4$ ) each SIL-2 ( $P = 0.01$ ). For this case, for a given demand, there are the following five possibilities:

- None of the SIFs fail
- One SIF fails
- Two SIFs fail
- Three SIFs fail
- All four SIFs fail

Each of the above is considered individually.

#### None of the SIF’s Fail

The probability of the SIF failing is 0.01. Therefore, the probability of it not failing is 1–0.01 or 0.99. For four SIFs, the probability of all four not failing (all work) is given by the multiplication rule above, or  $P(0 \text{ of } 4) = 0.99 \times 0.99 \times 0.99 \times 0.99 = 0.99^4$ .

#### One SIF Fails

If we label the four SIFs A, B, C, and D, and designate “1” as fail and “0” as not-fail, then the possibilities for one SIF to fail is given as follows:

A	B	C	D
1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Notice that there are four possible ways for one SIF to fail. This is given by the combination formula:

$$C_{Nk} = \frac{N!}{k! \cdot (N - k)!} \quad (1)$$

where  $C_{Nk}$  is the number of combinations of  $N$  objects taken  $k$  at a time without regard to order.

In this case,  $C_{41}$  is  $4!/(1! \times (4 - 1)!) = 4!/3! = 4$  possible combinations.

Examining row 1 of the above table, the probability that SIF A fails *and* SIFs B, C, and D not-fail is  $0.01 \times (0.99) \times (0.99) \times (0.99)$ . For the other rows, we may write as follows:

A	B	C	D	
0.01	0.99	0.99	0.99	$P_A = 0.01 \times (0.99)^3 = 9.7E-03$
0.99	0.01	0.99	0.99	$P_B = 0.01 \times (0.99)^3 = 9.7E-03$
0.99	0.99	0.01	0.99	$P_C = 0.01 \times (0.99)^3 = 9.7E-03$
0.99	0.99	0.99	0.01	$P_D = 0.01 \times (0.99)^3 = 9.7E-03$

The total probability of the four possible events (A fail/BCD not-fail) *or* (B fail/ACD not-fail) *or* (C fail/ABD not-fail) *or* (D fail/ABC not-fail) is the sum of the individual probabilities,  $P_A + P_B + P_C + P_D$  or  $4 \times (9.7E-03) = 0.0388$ . Thus, there is a 3.8% chance that, if we had four SIFs each SIL-2, exactly one would fail.

Notice that the probability of exactly one out four SIFs failing is given by:

$$P(1 \text{ of } 4) = C_{41} \times (0.01)^1 \times 0.99^{(4-1)}$$

that is, the probability of exactly one out of four failing is the probability of (one failing/three not-failing) times the number of ways of the combination (one failing/three not-failing) can occur.

#### Two SIFs Fail

For two SIFs, we follow the same procedure. The possible combinations of two failures are as follows:

A	B	C	D
1	1	0	0
1	0	1	0
1	0	0	1
0	1	1	0
0	1	0	1
0	0	1	1

There are six combinations of two SIFs failing given by  $C_{42} = 4!/(2! \times (4 - 2)!) = 4 \times 3 \times 2!/(2! \times 2) = 6$ .

The probability of obtaining row 1 is  $(0.01)^2 \times (0.99)^{4-2}$ . As there are six possible combinations and the probability of each combination is the same as row 1, the probability that exactly two out of four SIFs will fail is given by:

$$P(2 \text{ of } 4) = 6 \times (0.01)^2 \times (0.99)^{4-2} = 5.88\text{E-}04$$

### Three and Four SIFs Fail

For three SIFs failing, the number of possible combinations is  $4!/(3! \times (4 - 3)!)$  or 4 possible combinations. Therefore:

$$P(3 \text{ of } 4) = 4 \times (0.01)^3 \times (0.99)^{4-3} = 3.96\text{E-}06$$

For all four SIFs failing, there is only one possible combination and the probability is:

$$P(4 \text{ of } 4) = 1 \times 0.01^4 = 1.0\text{E-}08$$

### BINOMIAL PROBABILITY DISTRIBUTION

The above example may be generalized to any number of SIFs with any fixed probability of failure on demand as follows:

*Given  $N$  SIFs with probability of failure on demand,  $P$ , the probability that exactly  $k$  SIFs will fail is given by:*

$$P(k \text{ of } N) = C_{Nk} p^k (1 - p)^{N-k}$$

where  $C_{Nk} = N!/(k! \times (N - k)!)$  is the number of combinations of  $N$  objects taken  $k$  at a time (without regard to order).

The above equation is known as the Binomial Probability Distribution and is given in most textbooks on Statistics or Probability [2] as well as Perry's Handbook [3]. To be applicable, all of the SIFs must be completely independent and each must have the same probability of failure on demand.

SILs generally refer to a range of values for the probability of failure on demand. In practice, it is

expected that the higher value would be used for the probability of failure on demand in the above equation, which is conservative. For example, for SIL 1, the probability of failure on demand is greater than 0.01 but less than or equal to 0.1. Thus, for SIL 1, the probability of failure on demand,  $P$ , in the above equation would be 0.1. For SILs 2 and 3, the probability of failure on demand would be 0.01 and 0.001, respectively.

In Microsoft Excel, the function BINOMDIST( $k$ ,  $N$ ,  $p$ , False) calculates the probability that exactly  $k$  of  $N$  SIFs which will fail if the probability of an individual failure is  $P$ . For example, the probability that two of four SIFs will fail if each is SIL-2 ( $P = 0.01$ ) is given by BINOMDIST (2,4,0.01, False) = 5.88E-04 which agrees with the probability calculated earlier.

Tables 1 and 2 give the probabilities of exactly  $k$  of  $N$  SIFs failing for SIL-1 and SIL-2, respectively, and for  $N$  from 1 to 10. In each table, the solid line indicates the point at which the probability drops below 1E-03.

### PROBABILITY OF K OR MORE FAILURES

The binomial distribution gives the probability that exactly  $k$  of  $N$  SIFs will fail. For design, the quantity of interest is not the probability that exactly  $k$  of  $N$  SIFs will fail but rather the probability that  $k$  or more SIFs will fail. This is obtained by straightforward addition of the individual probabilities and may be summarized as follows:

*Given  $N$  SIFs with probability of failure on demand,  $P$ , the probability that  $k$  or more SIFs will fail is given by:*

$$P(k \text{ or more of } N) = P(k \text{ of } N) + P(k + 1 \text{ of } N) + P(k + 2 \text{ of } N) + \dots + P(N, N)$$

where the individual probabilities are obtained from the binomial probability distribution.

For example, for five SIFs with SIL-2, the probability that three or more SIFs fail may be obtained from Table 2 as follows:

$$P(3 \text{ or more of } 5) = P(3 \text{ of } 5) + P(4 \text{ of } 5) + P(5 \text{ of } 5) = 9.80\text{E-}06 + 4.95\text{E-}08 + 1.00\text{E-}10 = 9.85\text{E-}06$$

These calculations have been repeated for all combinations and the results are given in Tables 3 and 4 which give the probability of  $k$  or more of  $N$  SIFs will fail for SIL-1 and SIL-2, respectively.

### SYSTEM DESIGN

With Tables 3 and 4, it becomes relatively easy to decide how many SIF failures to consider in design. Consider for example, the design of a system with the following parameters:





Initiating event	Power failure
Frequency of Initiating Event	Once/10 years
Number of SIFs	6
SIL	2
Risk criteria	1 unacceptable event per 10,000 years

It should be noted that the allowable risk criteria given above is used for illustrative purposes only. In practice, the allowable risk criteria are determined by many factors not discussed in this article and must be given by Company Guidelines. Additional information on the allowable risk criteria can be obtained elsewhere [4].

If we decide to design the system for one SIF failing, then an unacceptable event would be two or more SIFs fail. From Table 4, the probability of two or more SIFs failing is 1.46E-03. The frequency with which the unacceptable event can be expected is:

$$\begin{aligned} & (\text{one demand}/10 \text{ years}) \times (1.46\text{E}-03) \\ & \text{unacceptable events/demand)} \\ & = 1.46\text{E}-04 \text{ unacceptable events/year} \end{aligned}$$

or 1.46 unacceptable events every 10,000 years. This frequency does not meet the risk criteria and we would need to design for two SIFs failing in which case an unacceptable event is three or more SIFs failing. From Table 4, the probability of three or more SIFs failing is 1.96E-05 and the frequency of an unacceptable event is (0.1)(1.96E-05) or approximately two events per 1,000,000 years. This easily meets the risk criteria and the design can proceed based on two SIFs failing.

The two SIFs which produce the largest load would typically be used for design. However, if the loads differ greatly in molecular weight, temperature, or heat of combustion, if the header geometry is complex, or if the set pressures of the individual valves differ, it may not be so easy to identify the two SIFs which should be used to design the header system. For this reason, it is best to consider all possible combinations of one and two SIFs failing to ensure the design is adequate. For this case, the number of possible combinations which would need to be examined is  $C_{61} + C_{62} = 6 + 15 = 21$  combinations.

#### CONSIDERATION OF LOADS AND SYSTEM GEOMETRY

The above methodology gives a precise answer to the question of determining the probability that  $k$  of  $N$  SIFs will fail on demand. However, the probability that  $k$  of  $N$  SIFs fail is not necessarily the same as the probability that some physical parameter of the system design (allowable backpressure, radiation) will be exceeded. The reason for this is that if the loads differ greatly then there will be some combinations of SIF failures in which the combined load is too small

to result in excessive backpressure/radiation even though that combination is included in the probability of  $k$  of  $N$  failures.

To see this, consider the system design example given earlier. Note that, if we designed the system for one SIF failure (in which case an unacceptable event is two or more SIF failures), then we could expect 1.46 unacceptable events per 10,000 years which just missed the criteria of 1.0 unacceptable events per 10,000 years. This then forced us to design for two SIF failures.

Now, as an extreme example, suppose the loads associated with the six SIFs were (1000, 1, 2, 3, 4, 5) in arbitrary units. In the design example, we could not design for one SIF failure because the probability of two or more failures exceeded the allowable risk criteria. In calculating the probability of failure of two or more SIFs, we had included all possible combinations of two failures— $C_{62}$ —or 15 combinations. However, if the system is designed for one SIF failure at 1,000 units, certain combinations of two failures, e.g., (1,2), (1,3), ... (2,3), (2,4)...(4,5), will not result in a system failure (excessive backpressure or radiation) and therefore did not need to have been considered in the probability of a system failure. In fact, there are only five combinations of two failures which could result in excessive backpressure or radiation—(1,000,1), (1,000,2)...(1,000,5). The probability of one of these combinations occurring is  $5 \times (0.01)^2(0.99)^{6-2}$  or 4.8E-04 system failures per demand. With an initiating event frequency of 1/10 years, a system failure can be expected (0.1)(4.84E-04) or 0.48 system failures every 10,000 years which meets the risk criteria. (If we add in the probabilities of three or more failures the answer is 0.49 system failures per 10,000 years). Thus by explicit consideration of the loads, we can design the system for one SIF failure rather than the two SIF failures which would have been required by straight application of the binomial theorem.

The above example may be summarized as follows:

Methodology	Binomial Theorem	Binominal Theorem	Explicit Consideration of Loads
Number of SIF failures allowed in design	One	Two	One
Unacceptable event	Two or more failures	Three or more failures	Two or more failures
Frequency of unacceptable event (events/year)	1.46E-04	1.96E-06	0.49E-04
Criteria (events/year)	1.00E-04	1.00E-04	1.00E-04
Meets criteria?	No	Yes	Yes



Although in this example, it was obvious that various combinations of loads were not controlling, in practice it is more difficult. As stated earlier, different loads, molecular weights, temperatures, set pressures, and heating values coupled with complex header geometry can make it extremely difficult to know what combinations will result in an unacceptable design. For this reason, a more systematic method of identifying system failures is required. This leads us to the following methodology:

*For a given relief header geometry with N SIFs, the probability of a system failure (excessive backpressure/radiation) is calculated as follows:*

1. Form a matrix of all possible combinations of  $k$  of  $N$  failures. Each row of the matrix represents a unique combination of SIF failures.
2. For each row, calculate the back pressures and radiation levels. If these are acceptable, put a "pass" in that row; otherwise put a "fail."
3. For each row with a "fail," calculate the probability of that specific row occurring. This is  $P(\text{specific row}) = p^k \times (1 - p)^{N-k}$ .
4. Determine the sum,  $P$ , of all the probabilities in the rows marked "fail."

The sum  $P$  is the probability of system failure (excessive backpressure or radiation) resulting from the initiating event.

For example, for 4 SIFs, SIL-2 with "1" indicating the specific SIF has failed and "0" indicating it has not-failed, the matrix of possible combinations of failures might look as follows:

SIF#	A	B	C	D	Calculation Result	Probability	Total for block $k$
$k = 1$	1	0	0	0	Pass		
	0	1	0	0	Pass		
	0	0	1	0	Pass		
	0	0	0	1	Pass		
$k = 2$	1	1	0	0	Pass		1.96E-04
	1	0	1	0	Fail	$(0.01)^2 \times (0.99)^{4-2}$	
	1	0	0	1	Fail	$(0.01)^2 \times (0.99)^{4-2}$	
	0	1	1	0	Pass		
	0	1	0	1	Pass		
	0	0	1	1	Pass		
$k = 3$	1	1	1	0	Fail	$(0.01)^3 \times (0.99)^{4-3}$	3.96E-06
	1	1	0	1	Fail	$(0.01)^3 \times (0.99)^{4-3}$	
	1	0	1	1	Fail	$(0.01)^3 \times (0.99)^{4-3}$	
	0	1	1	1	Fail	$(0.01)^3 \times (0.99)^{4-3}$	
$k = 4$	1	1	1	1	Fail	$(0.01)^4$	1.00E-08

The total probability of a system failure (excessive backpressure/radiation) is the sum of the probabilities in the above table or  $P(\text{system failure}) = 2.00\text{E-}4$

04. In practice, if radiation and excessive back pressure have different allowable risk criteria, then one would prepare two tables – one for radiation and one for back pressure.

Notice that in a block for a given  $k$ , if all the results are "fail," then the sum for the block may be taken directly for Table 1 or 2 depending on the SIL.

Notice also how fast the probability falls with increasing  $k$ . In the table above, the probabilities for  $k = 3$  and  $k = 4$  are so small that a calculation for the system is not required—it is sufficiently accurate to simply assume they all fail.

Generating the ones and zeros showing the possible combinations of failures is relatively easy on a spreadsheet. Automatic spreadsheet methods for generating the various combinations are available on the internet or may be obtained from the author.

#### SIMPLIFICATION FOR LARGE N

The rapid fall-off of probability with increasing  $k$  may be used to considerably reduce the number of calculations which must be done for a given system geometry. For example, for  $N = 10$ , the number of possible combinations  $C_{Nk}$  for each  $k$  may be calculated from Eq. 1 and is as follows:

$k$	$C_{Nk}$
1	10
2	45
3	120
4	210
5	252
6	210
7	120
8	45
9	10
10	1
Total	1,023

From the above, it would appear that 1,023 configurations would need to be evaluated. However, if the SIL is 2, then from Table 2 or 4, it is clear that for  $k = 3$  or more there is a negligible probability of the configuration occurring so that calculations would only need to be done only for  $k = 1$  and  $k = 2$  or 55 calculations. For  $k = 3, 4, \dots, 10$ , it would be conservative to assume that all of the SIFs fail with a combined probability (from Table 4) of  $1.14\text{E-}04$ . Similar simplifications will occur for any number of SIFs.

#### SUMMARY

In this article, two questions concerning relief header design have been addressed. For the first question, it is shown that, for  $N$  independent SIFs with a specified probability of failure on demand, the probability that  $k$  of  $N$  SIFs will fail is given by the binomial probability distribution. For the second question, which is given a header system geometry and fixed loads with  $N$  independent SIFs with a specified probability of failure on demand, it is shown that the probability of a system failure (excessive backpressure or

radiation) can be determined by identifying each possible configuration which will result in system failure, calculating the probability of occurrence for each of those specific configurations and then adding the resultant probabilities over all possible failures.

#### NOMENCLATURE

$N$  = number of independent SIFs

$P$  = probability of failure on demand (PFD)

$k$  = number of failures

$P(k \text{ of } N)$  = probability that  $k$  of  $N$  SIFs will fail

$C_{Nk}$  = number of combinations of  $N$  safety instrumented functions taken  $k$  at a time without regard to order

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